More Building on Taking the Derivative of an Integral

Recall FTC Part 1: If $g(x)=\int_a^x f(t)\,dt,$ then $g'(x)=f(x).$

Recall: $\frac{d}{dx}\int_1^x t^2\,dt$

Recall: when upper bound is more complex than x: $\frac{d}{dx}\int_1^{\sin x} t^2\,dt$

Recall: when constant is the upper bound: $\frac{d}{dx}\int_{\sin x}^1 t^2\,dt$

$\int_a^b f(x)\,dx=$

Do: $\frac{d}{dx}\int_{3x}^4 \left(7t^2-t+2\right)\,dt$

Remember, sometimes, it's advantageous to split the integral into two pieces.

Let's apply this to an FTC Pt 1 problem:

ex. $\dfrac{d}{dx}\int_{x^2}^{4x^2} \ln\sqrt{t}\ dt$

Let's streamline the previous problem using FTC Pt 2:

Revisit $\dfrac{d}{dx}\int_{x^2}^{4x^2} \ln\sqrt{t}\ dt$

Do: $\dfrac{d}{dx}\int_{x^3}^{\sin x} e^t\ dt$

ex: $\dfrac{d}{dx}\int_{e^{2x}}^{\sec^2 x} \sqrt{t}\ dt$

Another application using the splitting of integrals:

ex. Given $\int_3^5 f(x)dx = 17$ and $\int_5^8 f(x)dx = 12$, find $\int_3^8 f(x)dx$.

ex. Given $\int_0^{10} f(x)dx = 17$ and $\int_0^8 f(x)dx = 12$, find $\int_8^{10} f(x)dx$.

ex. Given $\int_0^{10} f(x)dx = 8$, find $\int_0^{10} \left(3f(x)+5\right)dx$.

Last Time: $\int \frac{\arctan x}{1+x^2}\,dx$

Compare: $\int \frac{x}{1+x^2}\,dx$

More Compare and Contrast:

Do: $\int \sqrt{1+x^2}\; x\,dx$

However: $\int \sqrt{1+x}\; x\,dx$